

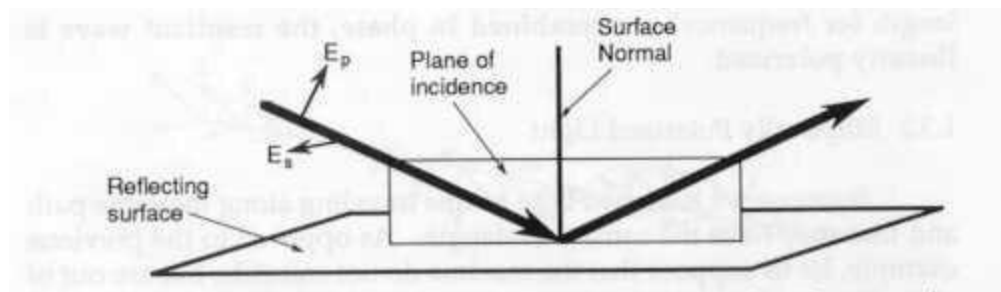
Ellissometria

**per il corso di Laboratorio di Ottica
Gigi Cristofolini, Nov 2007**

BASICS of ELLIPSOMETRY:

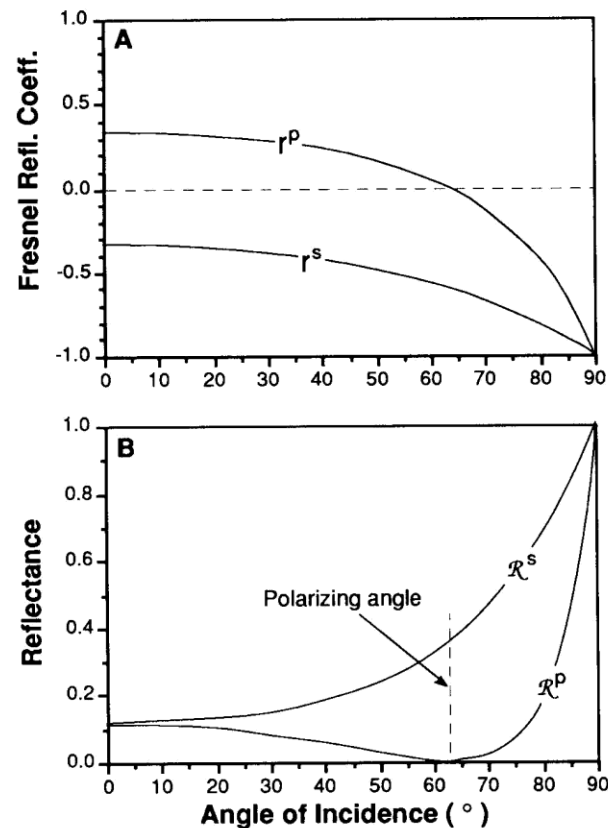
This technique is based on the **POLARIZING PROPERTIES** of reflection:

Reflection of light from a surface. The plane of incidence contains both incoming and outgoing beams, and the normal to the surface.






Top: reflected amplitude for p-polarized light r^p and s-polarized light r^s as a function of the angle of incidence Φ , note the zero crossing for r^p around $\Phi=63^\circ$.

Bottom: reflected intensities (e.g. reflectance) $R=|r|^2$ for p- and s-light respectively







2 Strategies:

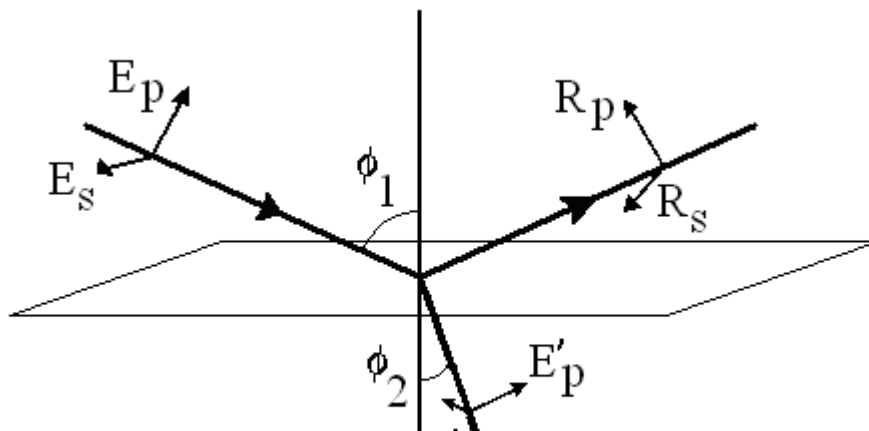
- **Null-ellipsometry:**

-  high precision,
-  good speed
-  but single wavelength measurements

- **Spectroscopic ellipsometry:**

-  measurement over large spectral range,
-  slow,
-  lower precision at each wavelength,
-  Thanks to Kramers-Kronig relations many more situations can be solved

Relazioni fondamentali



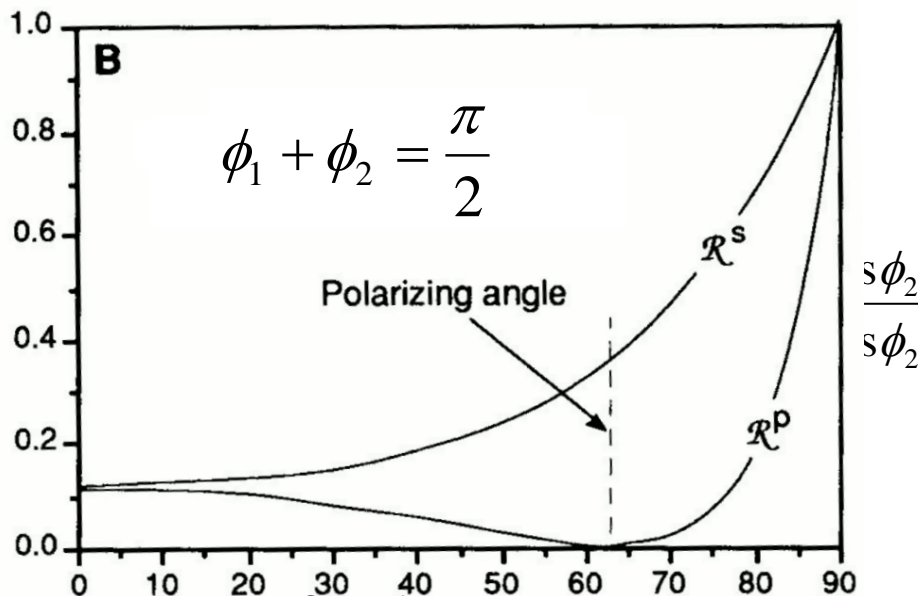
Coefficienti di

$$r_{12}^p = \frac{R_p}{E_p} = \frac{\tilde{N}_2}{\tilde{N}_1}$$

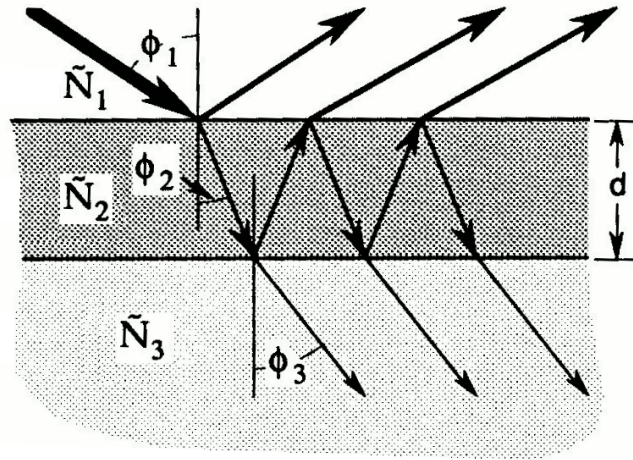
Legge di Snell

$$\tilde{N}_1 \sin \phi_1$$

Riflettanza



Riflessione a più interfacce



Lunghezza di fase del mezzo 2

$$\beta = 2\pi \left(\frac{d}{\lambda} \right) \tilde{N}_2 \cos^{-1} \phi_2$$

$$r^p = \frac{R_p}{E_p} = \frac{r_{12}^p + r_{23}^p e^{-i2\beta}}{1 + r_{12}^p r_{23}^p e^{-i2\beta}}$$

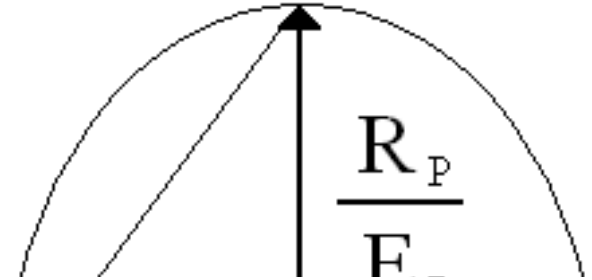
$$r^s = \frac{R_s}{E_s} = \frac{r_{12}^s + r_{23}^s e^{-i2\beta}}{1 + r_{12}^s r_{23}^s e^{-i2\beta}}$$

$$t^p = \frac{E'_p}{E_p} = \frac{t_{12}^p t_{23}^p e^{-i\beta}}{1 + t_{12}^p t_{23}^p e^{-i2\beta}}$$

$$t^s = \frac{E'_s}{E_s} = \frac{t_{12}^s t_{23}^s e^{-i\beta}}{1 + t_{12}^s t_{23}^s e^{-i2\beta}}$$

Ψ e Δ

$$E = \begin{pmatrix} |E_p| e^{i\delta_{Ep}} \\ |E_s| e^{i\delta_{Es}} \end{pmatrix} \quad R = \begin{pmatrix} |R_p| e^{i\delta_{Rp}} \\ |R_s| e^{i\delta_{Rs}} \end{pmatrix}$$

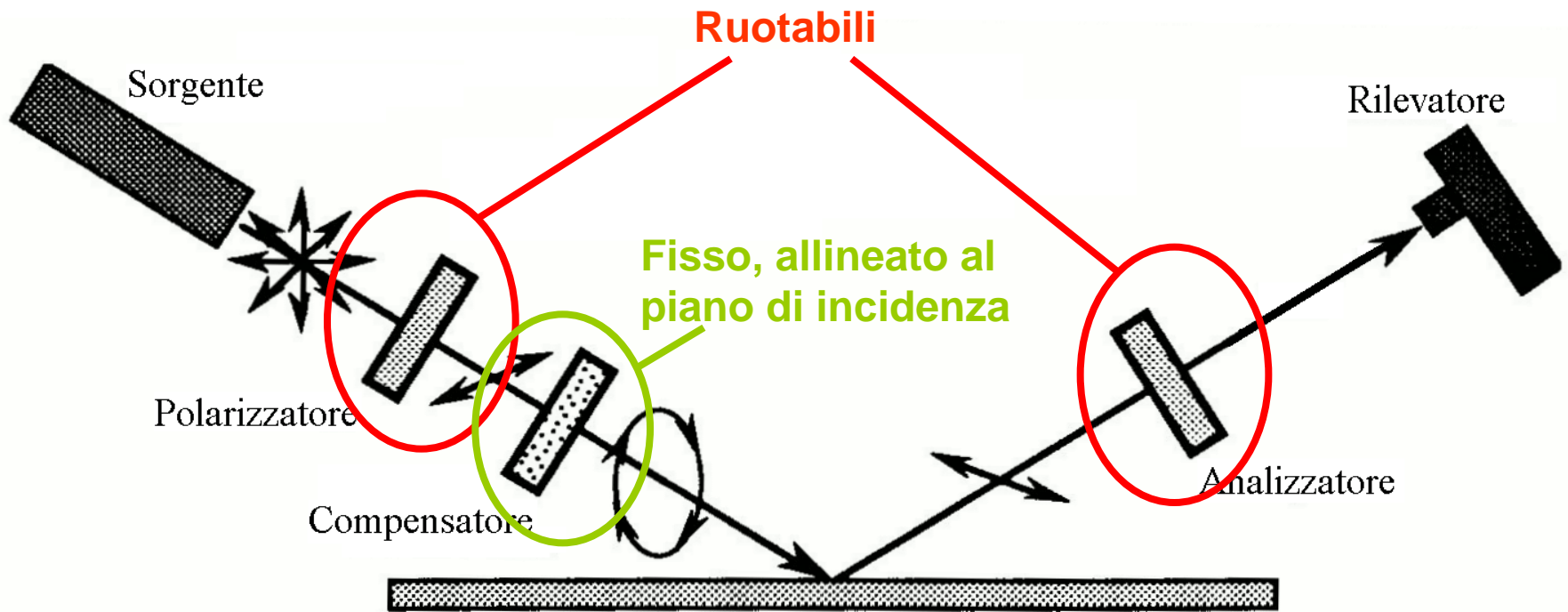


$$r^p = \frac{R_p}{E_p} = \frac{|R_p|}{|E_p|} e^{i(\delta_{Rp} - \delta_{Ep})} \quad r^s = \frac{R_s}{E_s} = \frac{|R_s|}{|E_s|} e^{i(\delta_{Rs} - \delta_{Es})}$$

Equazione fondamentale dell' ellissometria

$$\tan \Psi e^{i\Delta} = \frac{r^p}{r^s}$$

L' Ellissometro in configurazione PCSA:



Tra la Sorgente e il Polarizzatore è presente un' ulteriore lamina quarto d'onda che rende la radiazione in uscita dal laser perfettamente circolare, in modo che qualunque sia l' orientazione del polarizzatore

Luigi Cristofolini - Laboratorio di
Optica AA 2007-08 - Principi di

Ellissometria

Prima di procedere:

Polarization by Jones vector & matrices

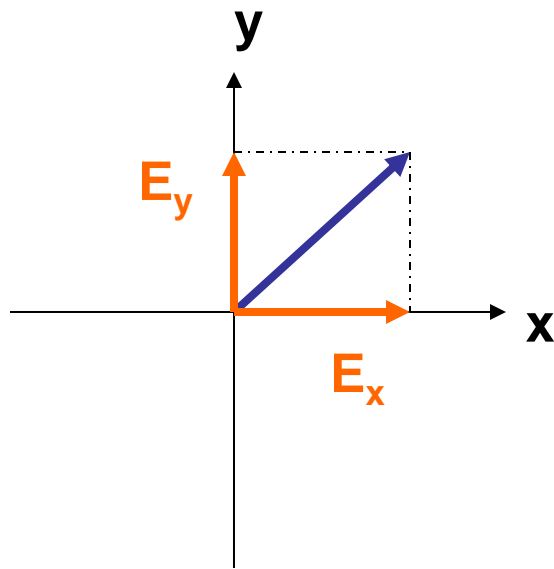
R. Clark Jones (1941)

PROs: simple, concise

CONs: only applicable to coherent, polarized waves

Matrix treatment of polarization

- Consider a light ray with an instantaneous E-vector as shown



$$\vec{E}(k, t) = \hat{i}E_x(k, t) + \hat{j}E_y(k, t)$$

$$E_x = E_{ox} e^{i(kz - \omega t + \varphi_x)}$$

$$E_y = E_{oy} e^{i(kz - \omega t + \varphi_y)}$$

Matrix treatment of polarization

- Combining the components

$$\vec{E} = \hat{i} E_{ox} e^{i(kz - \omega t + \varphi_x)} + \hat{j} E_{oy} e^{i(kz - \omega t + \varphi_y)}$$

$$\vec{E} = \left[\hat{i} E_{ox} e^{i\varphi_x} + \hat{j} E_{oy} e^{i\varphi_y} \right] e^{i(kz - \omega t)}$$

$$\vec{E} = \tilde{E}_o e^{i(kz - \omega t)}$$

- The terms in brackets represents the complex amplitude of the plane wave

Jones Vectors

- The state of polarization of light is determined by
 - the relative amplitudes (E_{ox} , E_{oy}) and,
 - the relative phases ($\delta = \varphi_y - \varphi_x$)of these components
- The complex amplitude is written as a two-element matrix, the Jones vector

$$\tilde{\mathbf{E}}_o = \begin{bmatrix} \tilde{E}_{ox} \\ \tilde{E}_{oy} \end{bmatrix} = \begin{bmatrix} E_{ox} e^{i\varphi_x} \\ E_{oy} e^{i\varphi_y} \end{bmatrix} = e^{i\varphi_x} \begin{bmatrix} E_{ox} \\ E_{oy} e^{i\delta} \end{bmatrix}$$

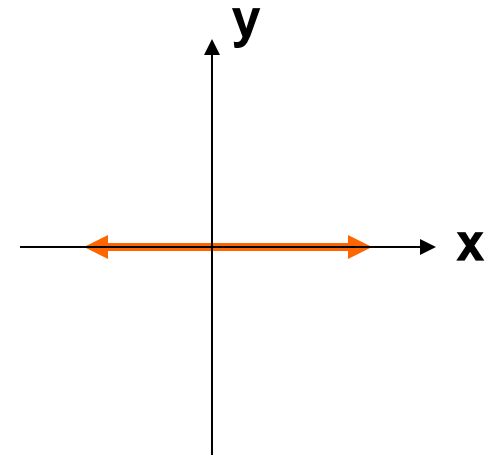
Jones vector: **Horizontally** polarized light

- The electric field oscillations are only along the **x**-axis
- The Jones vector is then written,

$$\tilde{\mathbf{E}}_o = \begin{bmatrix} \tilde{E}_{ox} \\ \tilde{E}_{oy} \end{bmatrix} = \begin{bmatrix} E_{ox} e^{i\varphi_x} \\ 0 \end{bmatrix} = \begin{bmatrix} A \\ 0 \end{bmatrix} = A \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

where we have set the phase $\varphi_x = 0$, for convenience

The arrows indicate the sense of movement as the beam **approaches** you



The normalized form is

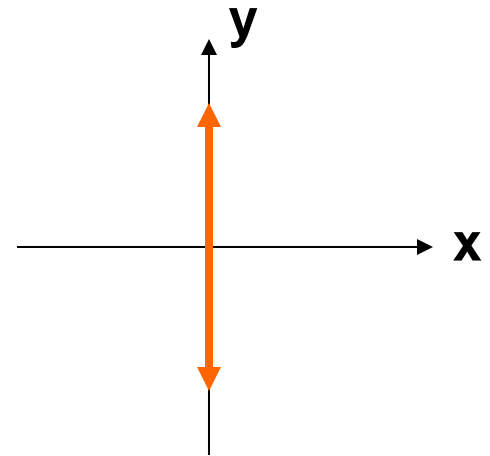
$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Jones vector: **Vertically** polarized light

- The electric field oscillations are only along the **y**-axis
- The Jones vector is then written,

$$\tilde{\mathbf{E}}_o = \begin{bmatrix} \tilde{E}_{ox} \\ \tilde{E}_{oy} \end{bmatrix} = \begin{bmatrix} 0 \\ E_{oy} e^{i\varphi_y} \end{bmatrix} = \begin{bmatrix} 0 \\ A \end{bmatrix} = A \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- Where we have set the phase $\varphi_y = 0$, for convenience



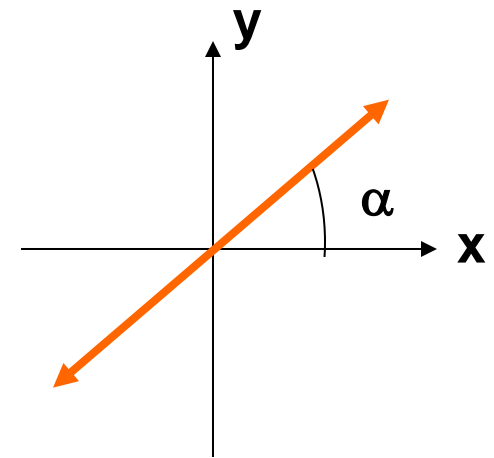
The normalized form is

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Jones vector: **Linearly** polarized light at an arbitrary angle

- If the phases are such that $\delta = m\pi$ for $m = 0, \pm 1, \pm 2, \pm 3, \dots$
- Then we must have,

$$\frac{E_x}{E_y} = (-1)^m \frac{E_{ox}}{E_{oy}}$$

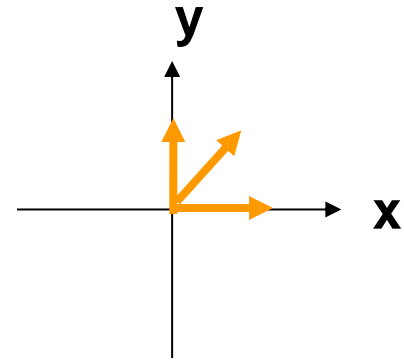


and the Jones vector is simply a line inclined at an angle $\alpha = \tan^{-1}(E_{oy}/E_{ox})$ since we can write

$$\tilde{E}_o = \begin{bmatrix} \tilde{E}_{ox} \\ \tilde{E}_{oy} \end{bmatrix} = A(-1)^m \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

Circular polarization

- Suppose $E_{ox} = E_{oy} = A$ and E_x leads E_y by $90^\circ = \pi/2$
- At the instant E_x reaches its maximum displacement ($+A$), E_y is zero
- A fourth of a period later, E_x is zero and $E_y = +A$



$$t=0, E_y = 0, E_x = +A$$

$$t=\tau/8, E_y = +A \sin 45^\circ, E_x = A \cos 45^\circ$$

$$t=\tau/4, E_y = +A, E_x = 0$$

Circular polarization

- The Jones vector for this case – where E_x leads E_y is

$$\tilde{E}_o = \begin{bmatrix} E_{ox} e^{i\varphi_x} \\ E_{oy} e^{i\varphi_y} \end{bmatrix} = \begin{bmatrix} A \\ Ae^{i\pi/2} \end{bmatrix} = A \begin{bmatrix} 1 \\ i \end{bmatrix}$$

- The normalized form is, $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$
- This vector represents circularly polarized light, where E rotates counterclockwise, viewed head-on
- This mode is called left-circularly polarized light
- What is the corresponding vector for right-circularly polarized light?

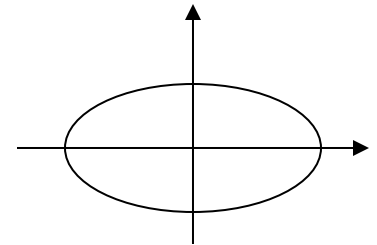
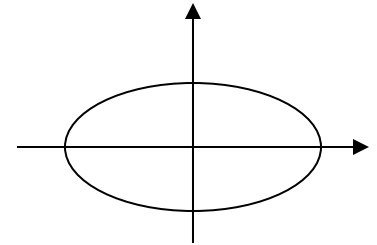
Replace $\pi/2$ with $-\pi/2$ to get $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$

Elliptically polarized light

- If $E_{ox} \neq E_{oy}$, e.g. if $E_{ox}=A$ and $E_{oy} = B$
- The Jones vector can be written

$$\begin{bmatrix} A \\ iB \end{bmatrix}$$

$$\begin{bmatrix} A \\ -iB \end{bmatrix}$$

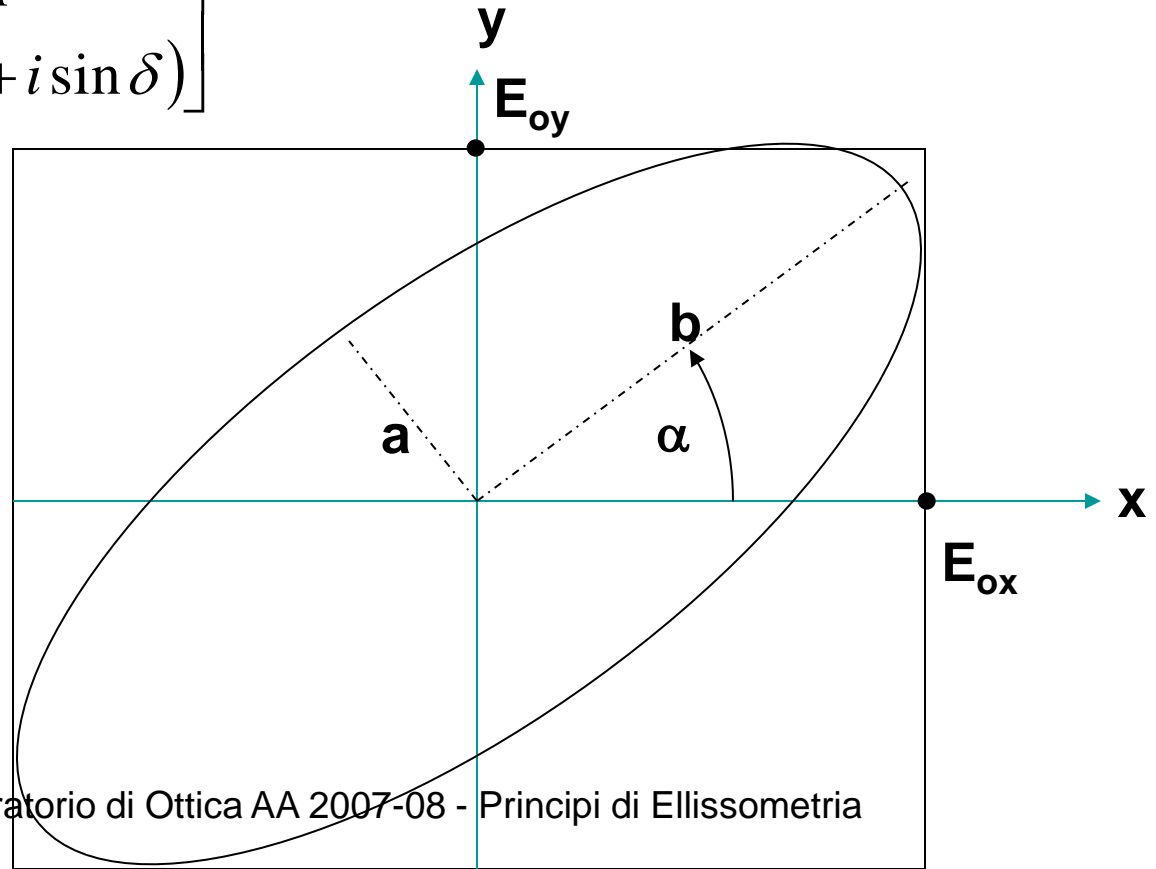


What determines the major or minor axes of the ellipse?

Jones vector and polarization

- In general, the Jones vector for the arbitrary case is an ellipse ($\delta \neq m\pi$; $\delta \neq (m+1/2)\pi$)

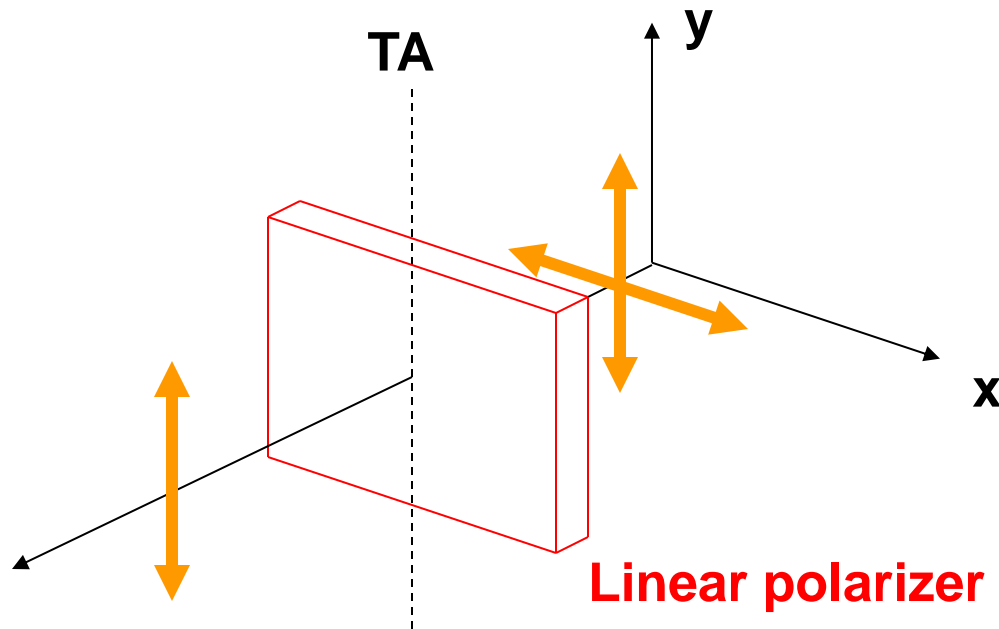
$$\tilde{\mathbf{E}}_o = \begin{bmatrix} E_{ox} \\ E_{oy} e^{i\delta} \end{bmatrix} = \begin{bmatrix} A \\ B(\cos\delta + i\sin\delta) \end{bmatrix}$$



$$\tan 2\alpha = \frac{2E_{ox}E_{oy} \cos\delta}{E_{ox}^2 - E_{oy}^2}$$

Optical elements: Linear polarizer

- Selectively removes all or most of the E-vibrations except in a given direction



Jones matrix for a linear polarizer

Consider a linear polarizer with transmission axis along the **vertical (y)**. Let a 2X2 matrix represent the polarizer operating on **vertically polarized** light.

The transmitted light must also be vertically polarized. Thus,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Operating on horizontally polarized light,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thus, $M = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ **Linear polarizer with TA vertical.**

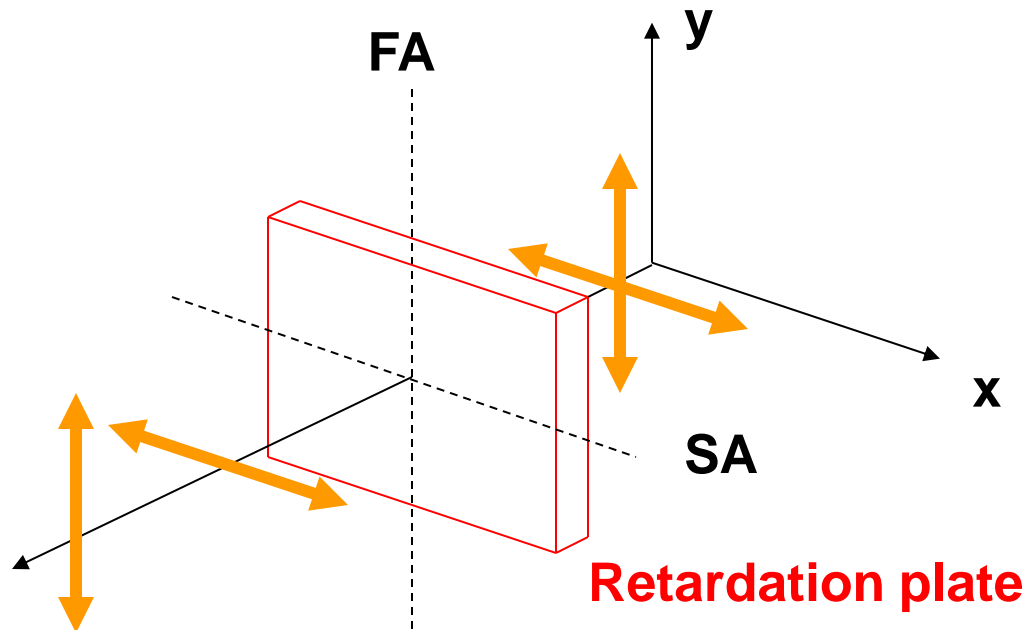
Jones matrix for a linear polarizer

- For a linear polarizer with a transmission axis at θ

$$M = \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix}$$

Optical elements: Phase retarder

- Introduces a phase difference ($\Delta\varphi$) between orthogonal components
- The fast axis (FA) and slow axis (SA) are shown



Jones matrix of a phase retarder

- We wish to find a matrix which will transform the elements as follows:
$$\begin{array}{l} E_{ox} e^{i\varphi_x} \quad \text{into} \quad E_{ox} e^{i(\varphi_x + \varepsilon_x)} \\ E_{oy} e^{i\varphi_y} \quad \text{into} \quad E_{oy} e^{i(\varphi_y + \varepsilon_y)} \end{array}$$

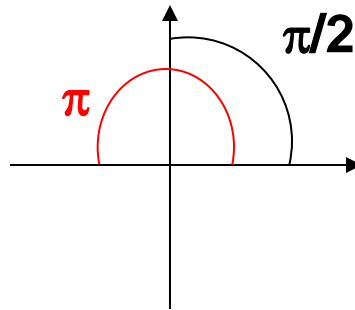
- It is easy to show by inspection that,

$$M = \begin{bmatrix} e^{i\varepsilon_x} & 0 \\ 0 & e^{i\varepsilon_y} \end{bmatrix}$$

- Here ε_x and ε_y represent the advance in phase of the components

Optical elements: Quarter/Half wave plate

- When the net phase difference
 $\Delta\varphi = \pi/2$: Quarter-wave plate
 $\Delta\varphi = \pi$: Half-wave plate



Jones matrix of a Quarter Wave Plate

- Consider a quarter wave plate for which $|\Delta\varepsilon| = \pi/2$
- For $\varepsilon_y - \varepsilon_x = \pi/2$ (Slow axis vertical)
- Let $\varepsilon_x = -\pi/4$ and $\varepsilon_y = \pi/4$
- The matrix representing a Quarter wave plate, with its slow axis vertical is,

$$M = \begin{bmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} = e^{-i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

Jones matrices: Half Wave Plate

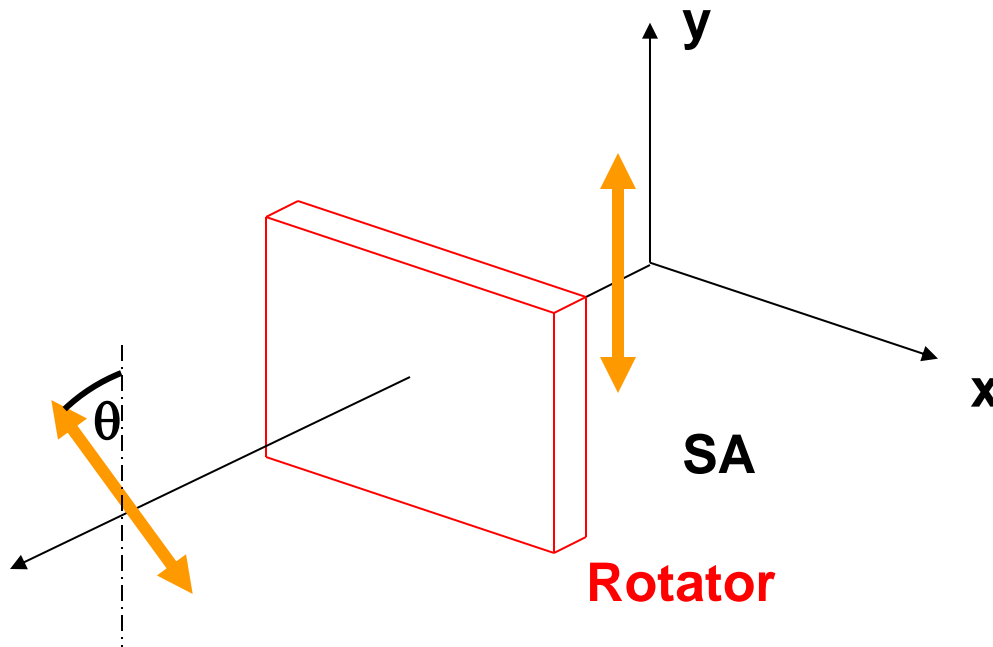
- For $|\Delta\varepsilon| = \pi$

$$M = \begin{bmatrix} e^{-i\pi/2} & 0 \\ 0 & e^{i\pi/2} \end{bmatrix} = e^{-i\pi/2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{HWP, SA vertical}$$

$$M = \begin{bmatrix} e^{i\pi/2} & 0 \\ 0 & e^{-i\pi/2} \end{bmatrix} = e^{i\pi/2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{HWP, SA horizontal}$$

Optical elements: Rotator

- Rotates the direction of linearly polarized light by a particular angle θ



Jones matrix for a rotator

- An E-vector oscillating linearly at θ is rotated by an angle β
- Thus, the light must be converted to one that oscillates linearly at $(\beta + \theta)$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} = \begin{bmatrix} \cos(\beta + \theta) \\ \sin(\beta + \theta) \end{bmatrix}$$

- One then finds

$$M = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix}$$

Summary of Jones Matrices

Optical element

Jones matrix

Horizontal linear polarizer

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Vertical linear polarizer

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

45° Linear polarizer

$$\frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

-45° Linear polarizer

$$\frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

Linear polarizer with axis at angle φ

$$\begin{pmatrix} \cos^2 \varphi & \cos \varphi \sin \varphi \\ \sin \varphi \cos \varphi & \sin^2 \varphi \end{pmatrix}$$

Left circular polarizer

$$\frac{1}{2} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$$

Right circular polarizer

$$\frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$$

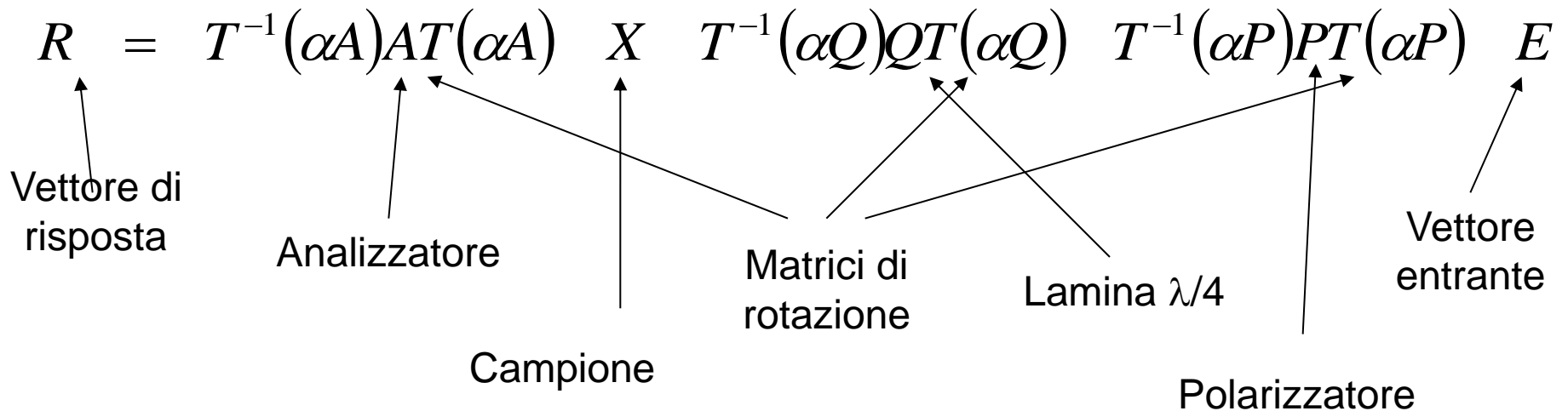
Half wave plate with horizontal fast axis

$$\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$$

Quarter wave plate with horizontal fast axis.

$$e^{i\pi/4} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

Con il formalismo delle matrici di Jones, calcoliamo la funzione di risposta dell'ellissometro:



$$RT(\alpha A) = \begin{pmatrix} |R| \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Gamma E(\Omega_1 + \Omega_2) \begin{cases} \Omega_1 = R_p \cos(\alpha A) [\cos(\alpha Q) \cos(\alpha Q - \alpha P) + i \sin(\alpha Q) \sin(\alpha Q - \alpha P)] \\ \Omega_2 = R_s \sin(\alpha A) [\sin(\alpha Q) \cos(\alpha Q - \alpha P) - i \cos(\alpha Q) \sin(\alpha Q - \alpha P)] \end{cases}$$

Cerchiamo la configurazione di "buio" $\rightarrow \Omega_1 + \Omega_2 = 0$

Funzione di risposta 2

$$\Omega_1 + \Omega_2 = 0 \rightarrow \frac{R_P}{R_S} = -\tan(\alpha A) \frac{\tan(\alpha Q) - i \tan(\alpha Q - \alpha P)}{1 + i \tan(\alpha Q) \tan(\alpha Q - \alpha P)}$$

Otteniamo una coppia di soluzioni:

$$(\alpha P_0; \alpha A_0) \quad \left(\alpha P_0 + \frac{\pi}{2}; \pi - \alpha A_0 \right)$$

Naturalmente girando i polarizzatori di 180 gradi otteniamo la stessa situazione, quindi esistono 16 soluzioni.

In generale si considera una sola coppia di soluzioni chiamata “misura in zona 1 e 3”.

Le corrispondenti soluzioni per $\alpha Q = +45^\circ$ sono chiamate “misura in zona 2 e 4”.

Introduzione degli errori

Considerando la possibilità che i componenti ottici siano imperfetti, inseriamo al posto di ciascuno di loro una matrice che corregga quella ideale per tenere conto del comportamento reale del componente, al di là della sua astrazione fin qui considerata:

- Viene introdotto un errore sulle rotazioni che alla generica matrice $T(\alpha)$ sostituisce $T(\alpha + \Delta\alpha)$
- Per descrivere le imperfezioni del compensatore inseriamo i parametri e e t_{1c} . Il primo rappresenta l'errore sul ritardo relativo indotto tra gli assi veloce e lento (quindi la lamina ora ritarda di t_{1c}), il secondo il leggero assorbimento della lamina (quindi il coefficiente di trasmissione della lamina da 1 diverrà $1 - t_{2c}$). Con queste ipotesi il componente diventa

$$W_Q Q = \frac{1}{1 - t_{2c}} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\left(\frac{\pi}{2} - t_{1c}\right)} \end{pmatrix}$$

Introduzione degli errori 2

- per descrivere le imperfezioni di usiamo un modello in cui il polarizzatore emette luce ellitticamente polarizzata, quindi t_{1P} rende conto dell' intensità trasmessa, sull' asse minore dell' ellisse, mentre t_{2P} della fase:

$$W_P P = \begin{pmatrix} 1 & 0 \\ 0 & t_{1P} + it_{2P} \end{pmatrix}$$

- l' errore campione considera eventuali errori di disallineamento. Questo varia l'angolo di incidenza e quindi la proiezione dell'ellitticità della radiazione incidente sul piano del campione e quella della radiazione uscente. Per tenere conto di questi effetti inseriamo due lamine di ritardo e prima e dopo il campione ad azimuth arbitrari :

$$W_X = \begin{pmatrix} 1 & 0 \\ 0 & e^{-it_{2X}} \end{pmatrix} \quad W'_X = \begin{pmatrix} 1 & 0 \\ 0 & e^{-it'_{2X}} \end{pmatrix}$$

Introduzione degli errori 3

- Risolvendo la funzione di risposta nel caso ideale, otteniamo sempre due coppie di soluzioni di soluzioni ($\alpha P_1; \alpha A_1$) e ($\alpha P_3; \alpha A_3$) nelle zone 1 e 3, che possiamo mediare per ottenere una coppia di valori Ψ e Δ , che nel caso ideale sarebbe

$$\Psi_{1,3} = -\frac{1}{2}(\alpha A_3 - \alpha A_1)$$
$$\Delta_{1,3} = (\alpha P_1 + \alpha P_3)$$

- Sostituendole nuove matrici che tengono conto degli errori e inserendo un eventuale errore di allineamento su tutti gli angoli la funzione di trasmissione cambia soluzioni, che differiscono dal caso ideale delle quantità:

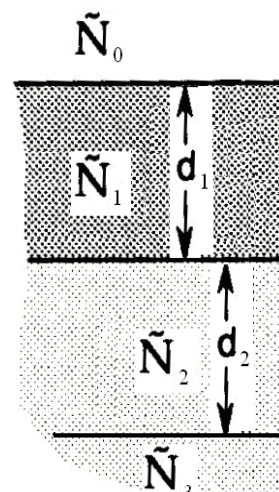
$$\delta\Psi_{1,3} = -t_{2P} \sin(2\Psi)$$

$$\delta\Delta_{1,3} = 2t_{1P} + 2\delta P - 2\delta Q + t_{2W_X} \cos(2\alpha W_X) + t_{2W'_X} \cos(2\alpha W'_X)$$

una media delle misure in zona 1 e 3 eliminano l'influenza di tutti gli errori da Ψ a parte l'ellitticità del Polarizzatore, mentre su Δ rimangono solamente l' assorbimento del Polarizzatore , l' errore sulle orientazioni del Polarizzatore e del Compensatore, e gli errori sull'allineamento del campione.

Interpretazione dei dati

Il modello utilizzato per interpretare i dati é una serie di strati ognuno caratterizzato da un indice di rifrazione e uno spessore.



Un programma apposito simula, a partire da un modello fornito dall'utente, le risposte del campione variando i parametri n e d dello strato selezionato in un range specificato dall'utente, simulando le coppie di valori Delta e Psi ottenute da ogni situazione, fino a trovare un setting compatibile coi valori misurati.

